Weekly Test solution Mathematics
TYJ – 02 TEST - 7

31. (b) If $(1-i)^n = 2^n$ ......(i)

We know that if two complex numbers are equal, their moduli must also be equal, therefore from (i), we have

$|(1-i)^n| = |2^n| \Rightarrow |1-i|^n = |2|^n$, $(\because 2^n > 0)$

$\Rightarrow \left[\sqrt{1^2+(-1)^2}\right]^n = 2^n \Rightarrow (\sqrt{2})^n = 2^n$

$\Rightarrow\ 2^{n/2} = 2^n \Rightarrow \frac{n}{2} = n \Rightarrow n = 0$

Trick : By inspection, $(1-i)^0 = 2^0 \ \Rightarrow 1 = 1$

32. (d) $(1+i)^5(1-i)^5 = (1-i^2)^5 = 2^5 = 32$.

33. (b) $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{2} = \frac{2i}{2} = i$

$\therefore \left(\frac{1+i}{1-i}\right)^m = i^m = 1$ (as given)

So the least value of $m = 4$ $\{\because i^4 = 1\}$

35. (c) $\frac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1-2i\sin\theta)(1+2i\sin\theta)} = \left(\frac{3-4\sin^2\theta}{1+4\sin^2\theta}\right) + i\left(\frac{8\sin\theta}{1+4\sin^2\theta}\right)$

Now, since it is real, therefore Im $(z) = 0$

$\Rightarrow \frac{8\sin\theta}{1+4\sin^2\theta} = 0 \Rightarrow \sin\theta = 0$, $\therefore \theta = n\pi$

where $n = 0, 1, 2, 3, ...$

Trick : Check for (a), if $n = 0, \theta = 0$ the given number is absolutely real but (c) also satisfies this condition and in (a) and (c), (c) is most general value of θ.

36. (c) Equation $(x+iy)(2-3i) = 4+i$

$\Rightarrow (2x+3y) + i(-3x+2y) = 4+i$

Equating real and imaginary parts, we get

$2x + 3y = 4$ ......(i)

$-3x + 2y = 1$ ......(ii)

From (i) and (ii), we get $x = \frac{5}{13}, y = \frac{14}{13}$

Aliter : $x + iy = \frac{4+i}{2-3i} = \frac{(4+i)(2+3i)}{13} = \frac{5}{13} + \frac{14}{13}i$.

37. (b) $a+ib > c+id$, it is defined if and only if imaginary parts must be equal to zero.

Therefore $ib = id = 0 \Rightarrow b = d = 0$ $(\because i \neq 0)$

38. (c) $\sum_{k=0}^{100} i^k = x+iy, \Rightarrow 1 + i + i^2 + ... + i^{100} = x+iy$

Given series is G.P.

$\Rightarrow \frac{1.(1-i^{101})}{1-i} = x+iy \Rightarrow \frac{1-i}{1-i} = x+iy$

$\Rightarrow 1 + 0i = x+iy$

Equating real and imaginary parts, we get the required result.

41. (c) $|z| - z = 1+2i$

Let $z = x+iy$, therefore $|x+iy| - (x+iy) = 1+2i$

Equating real and imaginary parts, we get

$\sqrt{x^2+y^2} - x = 1$ and $y = -2 \Rightarrow x = \frac{3}{2}$

Hence complex number $z = \frac{3}{2} - 2i$.

Trick : Since $\left|\frac{3}{2} - 2i\right| - \left(\frac{3}{2} - 2i\right)$

$= \sqrt{\frac{9}{4} + 4} - \frac{3}{2} + 2i = \frac{5}{2} - \frac{3}{2} + 2i = 1 + 2i$

42. (c) $|z| = 4$ and $arg\, z = \frac{5\pi}{6} = 150^o$

Let $z = x + iy$, then $|z| = r = \sqrt{x^2 + y^2} = 4$

and $\theta = \frac{5\pi}{6} = 150^o$

$\therefore\ x = r\cos\theta = 4\cos 150^o = -2\sqrt{3}$.

and $y = r\sin\theta = 4\ \sin 150^o = 4\frac{1}{2} = 2$

$\therefore\ z = x + iy = -2\sqrt{3} + 2i$

Trick : Since $arg\, z = \frac{5\pi}{6} = 150^o$, here the complex number must lie in second quadrant, so (a) and (b) rejected. Also $|z| = 4$ which satisfies (c) only.

43. (c) If $z = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}} = \frac{(1 - i\sqrt{3})(1 - i\sqrt{3})}{(1 + i\sqrt{3})(1 - i\sqrt{3})}$

$= \frac{1 - 3 - 2i\sqrt{3}}{1 + 3} = \frac{-2 - 2i\sqrt{3}}{4} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

Thus $arg(z) = \tan^{-1}\frac{y}{x} = \tan^{-1}\sqrt{3} = \frac{\pi}{3} = 60^{o.}$

Since the complex number lies in III quadrant, therefore

$arg(z)$ is $180^o + 60^o = 240^o$

Aliter : $arg\left(\frac{1 - i\sqrt{3}}{1 + i\sqrt{3}}\right) = arg(1 - i\sqrt{3}) - arg(1 + i\sqrt{3})$

44. (c) $arg\left(\frac{3 + i}{2 - i} + \frac{3 - i}{2 + i}\right) = arg\left(\frac{6 + 5i + i^2 + 6 - 5i + i^2}{5}\right)$

$= arg\left(\frac{10}{5}\right) = 0$.